Frequently, the technological tools used in specific sociocultural contexts unleash the development and elaboration of mathematical models (Rosa and Orey 2015). This curriculum model provides educators with a "critical way, with the communicative, analytical, and technological instruments necessary for life in the 21st century" (D'Ambrosio and D'Ambrosio 2013, p. 22). The incorporation of the trivium into classrooms implies a curricular reconceptualization in which ethnomathematics and mathematical modelling are tools for pedagogical action.

3.3.5 Some Considerations

Currently, there is growing awareness about the understanding and comprehension of mathematical ideas, procedures, and practices developed by the members of distinct cultural groups. This is primarily due to the expansion of studies related to culture, history, anthropology, linguistics, and ethnomathematics. Discoveries made through the ongoing investigations of the many and diverse theoretical and empirical studies show that it is possible to internationalize mathematical practices that emerge in different cultural contexts.

An important characteristic of ethnomathematics is the transformational power it has to help rethink the nature of mathematics. This means that one possible purpose for ethnomathematical studies and their innovative approaches could be to foster the development or transformation of mathematics. We hope that these innovative approaches of ethnomathematics will provide a better understanding of this research program.

3.4 Polysemic Interactions between Ethnomathematics and Culturally Relevant Pedagogy

Milton Rosa and Maria Elena Gavarrete

Over the past three decades, the theoretical bases for ethnomathematics (D'Ambrosio 1985) and culturally relevant pedagogy (Ladson-Billings 1995) have sought to ease sociocultural concerns as part of an examination of the cultural and socioeconomic influences on the processes related to teaching and learning mathematics. This includes knowledge, as well as a commitment to challenging social injustice, and reflections upon educational challenges involving identifying obvious and subtle individual, institutional, and cultural actions that perpetuate social structures.

The overall goal of this theoretical base is to empower students through learning activities that help them develop literacy, numeracy, and technological, social, and political skills in order to be active participants in a democratic society. It is important to emphasize that investigations conducted regarding culturally relevant

pedagogy study the cultural congruence between the backgrounds of students, communities, and schools, which in turn form one of the main principles of an ethnomathematics program (Hart 2003).

This means that cultural congruence indicates the respect teachers have for the social, cultural, and linguistic backgrounds of students. Educators (both the school management and teachers) must gain knowledge of and develop respect for the diverse cultural traditions, languages, prior mathematical knowledge, and respective community contexts of their students so that they are able to implement the principle of cultural congruence in schools and classrooms.

Mathematics tends to be presented as a set of objective and universal facts and rules; it is commonly viewed as *culture free* and not considered a socially and culturally constructed discipline. In order to change this perception, it is necessary that curriculum developers and teachers take into account what is considered mathematics and how this knowledge relates to the norms and values of diverse cultures (Rosa 2014). If, as educators, we come to integrate the diverse cultures we encounter in our school communities, then there is a need to create a conceptual framework to make coherent decisions regarding curricular activities concerning the mathematics curriculum.

3.4.1 Ethnomathematics and Culturally Relevant Pedagogies in Teacher Education

An important change in mathematical instruction needs to accommodate continuous and ongoing changes in students' demographics in mathematics classrooms around the world. Since ethnomathematics proposes that educators contextualize their mathematics teaching and learning by relating mathematical content to the sociocultural experiences of their students, it has become necessary to integrate culturally relevant pedagogies and diverse ethnomathematics perspectives into existing teacher education programs.

The guidelines of the National Council of Teachers of Mathematics (NCTM 1991), the Brazilian Ministry of Education and Culture (Brasil 1997), and the Ministry of Public Education (Costa Rica 2012) highlight the importance of building connections between mathematics and the sociocultural contexts of students. Thus, when students are encouraged to examine mathematical activities in their own sociocultural contexts, they realize that mathematics procedures and practices are not trivial, as they see them connected to their daily lives (Rosa and Orey 2007). In this perspective, students may succeed in mathematics when their understanding of it is linked to real and meaningful cultural referents and when instruction assumes that all students are capable of mastering mathematics (Ladson-Billings 1995).

Curricular activities developed according to principles of culturally relevant pedagogy focus on the role of mathematics in sociocultural contexts. These

activities involve ideas and procedures associated with ethnomathematical perspectives to solve problems. In this approach *ethno-* is defined as a culturally identifiable group with its own jargon, codes, symbols, myths, and even specific ways of reasoning and inferring; *mathema* is defined as categories of analysis; and tics is defined as methods or techniques for solving daily problems (Rosa and Orey 2013).

The inclusion of cultural aspects in a mathematics curriculum has long-term benefits for students' mathematical achievements, as these aspects contribute to the perception that mathematics is part of our daily lives and deepen the understanding of its nature by enhancing students' ability to make meaningful connections. Pedagogical work towards an ethnomathematics perspective allows for a broader analysis of school contexts in which pedagogical practices transcend classroom environments (Rosa 2014).

Ethnomathematics presents possibilities for educational initiatives and new curriculum objectives based on culturally relevant pedagogies. However, one dilemma regarding this issue is how to prepare teachers to create curriculum activities based on culturally relevant pedagogies and ethnomathematics (Greer 2013). One approach to solve this dilemma is to focus on the importance of promoting the dissemination of heritage aspects of local, cultural, and mathematical knowledge in order to help students strengthen their own cultural identities in school environments (Gavarrete 2014). It then becomes necessary to list a number of assumptions that prevail in mathematics education regarding Eurocentric perceptions found in the development of mathematics.

This approach shows that there is a need to encourage reflection about the development of mathematics in distinct cultures as well as the need to include ethnomathematics in teacher training processes in order to elevate pedagogical actions and combat the exclusion promoted by monocultural perspectives in the mathematics curriculum. Educational monocultural perspectives distort or limit the educational process of ethnomathematics, especially in indigenous contexts, because textbooks and other curricular materials have given little attention to non-representant, non-mainstream cultures in rural areas (Gavarrete 2012).

For example, it is necessary to study the mathematical knowledge developed by the members of Ngäbes, Bribris, and Cabécares ethnic groups in Costa Rica in order to propose pedagogical actions for teacher education programs pertaining to their worldview and particular logical traditions. The respect for the ideas and procedures implicit in social practices will provide prospective teachers with pedagogical tools that help them conduct teaching and learning processes in a contextualized fashion (Gavarrete 2012).

In this regard, it is necessary to propose a discussion about cultural relevance in teacher education programs in order to help prospective teachers acknowledge the relationship between cultural and school mathematical knowledge. This approach also fosters a reflective attitude about the universality and contextualization of mathematical knowledge, since pedagogical work with ethnomathematics promotes teacher creativity when developing a mathematics curriculum that is connected to the sociocultural environment of the students.

3.4.2 Culturally Relevant Activities Based on Ethnomathematics: Curricular Implications

Ethnomathematical approaches are intended to make school mathematics more relevant and meaningful to students in order to increase the overall quality of education and assert more culturally relevant views of mathematics. The application of any culturally relevant pedagogies and accompanying ethnomathematical perspectives in classrooms validates and incorporates the cultural background resulting from the ethnic heritage of students as well as their current interests into the daily instructional practices of teachers. It empowers students intellectually, socially, emotionally, and politically by using their sociocultural and historical realities and contexts to convey knowledge, impart academic skills, and change students' attitudes towards academic instruction (Ladson-Billings 1995).

This pedagogical approach is achieved through dialogue when community members, teachers, and students discuss mathematical themes that help them work on problems that are directly relevant to their community. In this context, students investigate conceptions, traditions, and mathematical practices developed by members of distinct cultural groups in order to incorporate them into the mathematics curriculum. Below we share two examples that demonstrate the kind of development found in culturally relevant activities in order to explore ethnomathematical relationships between ideas, procedures, and practices developed by the members of distinct communities.

(a) Measuring land

The use of the practice of cubic content calculation (cubação) as a pedagogical proposal to elaborate activities for the teaching and learning of mathematics shows the importance of the contextualization of problems in the learning environment through the connection of ethnomathematics and culturally relevant pedagogy concepts. In this regard, Knijnik (1997) proposed activities related to the demarcation of land from research work with the members of Movimento dos Trabalhadores Rurais Sem Terra (MST)[2] in Southern Brazil.

The land demarcation activity involved a method for calculating cubic content, which is a traditional mathematical practice applied by the members of this movement. It concerns the solution of problems of measuring irregular-shaped land. Figure 3.1 shows how to calculate an area of land with a quadrilateral shape measuring 114 m × 152 m × 90 m × 124 m.

Thus, the mathematical knowledge developed by the members of Movimento dos Trabalhadores Rurais Sem Terra presents a model that transforms the shape of the given plot of land into a rectangle (Fig. 3.2) whose dimensions are 138 m by 102 m with an area of 14,076 m^2.

[2]Movimento dos Trabalhadores Rurais Sem Terra (MST), the "Landless Rural Workers Movement," is a social movement in Brazil whose purpose is to fight for the redistribution of land to poor workers in general.

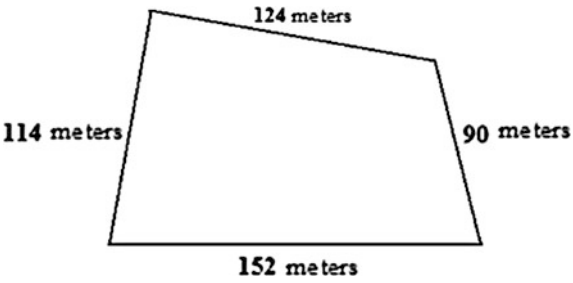

Fig. 3.1 Land with an *irregular quadrilateral shape*

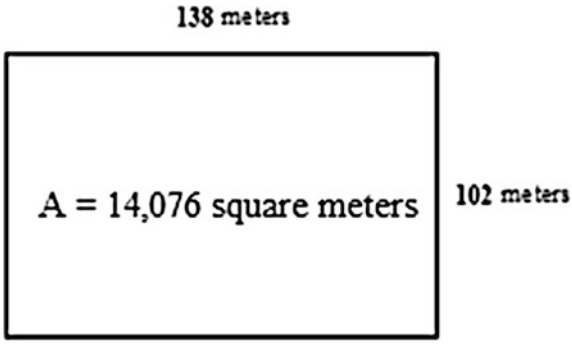

Fig. 3.2 Land with a *regular rectangular shape*

The following model explains this mathematical practice:

- Transform the irregular quadrilateral shape into a rectangle whose area can be determined through the application of the formula $\boldsymbol{A = b \cdot h}$.
- Determine the dimensions of the rectangle by calculating the mean of the two opposite sides of the irregular quadrilateral.

$$\text{Base} = \frac{152 + 124}{2} = 138\,\text{m}$$
$$\text{Height} = \frac{114 + 90}{2} = 102\,\text{m}$$

- In order to find the area of this irregular quadrilateral, it is necessary to determine the area of the rectangle.

$$A = b \cdot h$$
$$A = 138 \cdot 102$$
$$A = 14{,}076\ \text{m}^2$$

There is yet another model developed by the members of Movimento dos Trabalhadores Rurais Sem Terra that explains this mathematical practice. The members of this cultural group transform the irregularly shaped quadrilateral into a

square with 120-m sides, which has an area of 14,400 m^2. Thus, the measure of 120 m was calculated by adding the dimensions of the quadrilateral and then dividing it by four, which is the number of sides of the irregular quadrilateral (Rosa and Orey 2013).

(b) Mathematization of the tipi

Spatial geometry is inherent to the shape of the tipi and it was used to remind, indeed symbolize, the universe in which the Plains People lived. The Sioux language word tipi refers to a conical skin tent common among these North American tribes. Through history, the nomadic prairie people observed that the tripodal foundation appeared to be perfectly adapted for harsh environments.

There is evidence that Sioux people had an understanding of the characteristics and properties of geometry concepts such as triangles. For example, the majority of Sioux tribes use a three-pole foundation (Fig. 3.3) because it is stronger and offers a firmer foundation for the tipi (Rosa and Orey 2013).

The base of the tipi formed by the three-pole foundation is the triangle ABC (Fig. 3.4). The midpoints of each of the sides of ΔABC are points M, N, and P. It is possible to match each vertex of ΔABC to the midpoint of each opposite side, which gives us the straight lines AM, BN, and CP.

These straight lines form three medians, which are the straight lines connecting the midpoint of each opposite side of the triangle to its vertex. The medians intersect at one single point called the centroid. Archimedes demonstrated that the medians of a triangle meet at its balance point or center of gravity, which is the centroid of the triangle. Native Americans place their fire and altar at this point in the Tipi because it "holds a definite power and holiness" (Orey 2000, p. 246).

(c) Classification as a mathematical activity in indigenous contexts

Classification is an essential activity in some indigenous communities. Examples of cultural mathematical knowledge (CMK) show that indigenous people possess a holistic sense of reality that allows them to establish a conceptual structure in which relationships between the world and its tangible and intangible objects are understood, connected, and represented. Therefore, CMK is associated with classification systems practiced through oral tradition, and it uses metaphorical and symbolic logic that is applied to the study of ethnic groups whose cosmogony includes other representational systems such as the physical and symbolic mythical worlds (Gavarrete 2015).

For example, classification of linguistic systems of three Chibchan indigenous groups has been studied linguistically within an ethnomathematical perspective in Costa Rica. These oral indigenous languages use numeric classification to identify objects that are arranged or counted according to geometric shapes used in their daily lives, such as long, flat, and round (Gavarrete 2015).

In the particular case of the Cabecar language, the numeric metaphor sa'jula refers to the five fingers of a hand to represent the number five. Thus, in general,

Fig. 3.3 Three-pole foundation of the tipi

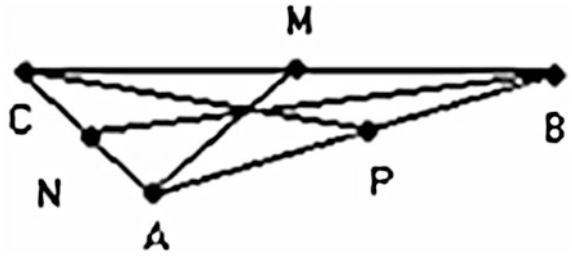

Fig. 3.4 Triangle *ABC*, which forms the base of the tipi

multiples of five relate to flat classification because the palm of the hand is flat, but when numbers are not multiples of five or objects have shapes other than flat, a combination of numeric classifications is used (Gavarrete 2012).

This specificity regarding classification systems in the indigenous communities of Costa Rica demands a reflection on the lack of contextualized academic resources and about the pedagogical challenge of teaching arithmetic in a culture that applies diverse systems of numeric classification that differ from academic mathematics used in schools (Gavarrete 2013).

3.4.3 Some Remarks

The objective of developing a mathematical curriculum based on ethnomathematics and culturally relevant pedagogies is to help students become aware of how people mathematize and think mathematically in their own cultures and to use this awareness to learn about formal mathematics, and increase their ability to mathematize in any context in the future. Students also come to value and appreciate their previous mathematical knowledge, which allows them to understand and experience these cultural activities from a mathematical point of view, thereby allowing them to make the link between school mathematics and the real world. An ethnomathematical curriculum helps students understand the nature of mathematics because it presents them with an effective tool that can contribute to their learning of mathematics (Rosa and Orey 2007).

The integration of ethnomathematics and culturally relevant pedagogies into the mathematics curriculum focuses on the development of this research area as a process rather than a collection of facts, as it is based on the idea that mathematics is a human creation that has emerged as people attempt to understand and comprehend the world around them. Therefore, mathematics can be seen as a rich and diverse cultural process and a human activity rather than just a set of academic content (Rosa 2014).

Mathematics knowledge in the context of a culturally relevant pedagogy is perceived in an ethnomathematical perspective because teachers build from the students' informal mathematics and direct the lesson toward their culture and experiences while developing their critical thinking skills. This environment allows us to reflect on the nature of mathematics, culture, education, and society and the relationships among them in order to include pedagogical practices in the teaching and learning of mathematics that address deeper notions of equality (Gavarrete 2014).

Therefore, we reiterate the importance of promoting a sociocultural approach in the mathematics curriculum in order to combat the curricular decontextualization resulting from a monocultural view. However, this approach faces the challenge of transcending ethnocentrism and enriching the process of teaching and learning mathematics through the incorporation of ethnomathematics and culturally relevant pedagogy into teacher education programs.