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Characterization and development of citizenship competence in the training of secondary school mathematics teachers

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Abstract. This paper is intended to determine the knowledge of teachers during their period of training of what characterizes a good citizen, to shed light on the concept of citizenship through activities related to the teaching of statistics and probability, and on how this concept may be developed in secondary education. These activities were implemented during the first half of 2014 in a group of future mathematics teachers pursuing Bachelor?s and Licenciatura degrees in Mathematics Teaching at the Universidad Nacional de Costa Rica. Trainee teachers showed significant conceptual weaknesses which hindered their ability to create more sophisticated activities for teaching this subject matter and promoting citizenship in high school students.

KeyWords: mathematics education, teacher training, competencies, citizenship

1.1 Introduction

As part of the social particularities of previous decades, education has developed in a complex and not always consistent way. Historically, teaching and learning activities were focused on contents in which the separation of educational subjects and activities was emphasized.

In addition, the adoption of standardized tests as preferred assessment instruments has possibly contributed to the creation of an important gap between content learning and the formation of citizens. This means that, in some cases, more attention is given to content rather than to the formation that takes place when a subject matter is understood.

This problem is relevant when considering education in mathematics. Formal education offered in this discipline should not be limited only to thematic concepts or elements, but should also prepare citizens to confront problems in their surroundings in an efficient and responsible manner.

Therefore, part of the current needs for developing effective mathematics education consists of determining ways to reduce this gap, guaranteeing that this education has a defined component of formation of citizens. To undertake this task, it is essential to know the type of formation desired, and to have ways for creating practical activities involving citizenship competencies.

This document addresses the subject of citizenship as a proposal for interpreting citizenship in education, specifically in mathematics education.

One way to diagnose part of the situation is to study trainee mathematics teachers and their knowledge and attitudes about the concept of citizenship. This article is therefore intended to determine the knowledge of trainee teachers about the characteristics of a good citizen, to shed some light on the concept of citizenship through activities related to the teaching of statistics and probabilities, and on how this concept may be developed in secondary education.

1.2 Theoretical framework

The concept of citizenship

To understand the concept of citizenship, it is useful to consider interpretations of citizenship by various authors in the Costa Rican context as examples.

The term *citizenship*is a polysemic concept; it has different definitions, and links various elements, depending on the context in which it is used. In some cases, it is related to politics and democracy; in other environments, it is associated with ethics and moral values; but the concept is most commonly related to the characteristics of a *good* individual in *society*. In this regard, , Marín (2006, , cited by Alfaro and Badilla, 2013 [1, pag 50]) states that "in Costa Rica we make a terrible jumble of tendencies and traditions" , when referring to citizenship, using the term interchangeably to mean that individuals abide by the law and and respect rights, or to refer to moral education, male chauvinism and feminism, gender and values, among other topics.

Another fundamental issue to consider when trying to understand the concept is that the Costa Rican educational system has somewhat biased contents, and a poorly integrated perspective on the issues of citizenship addressed in various courses has resulted in situations such as *citizenship* being traditionally addressed as part of social studies courses in elementary education, and as part of civic education in high school.

For Montoya [5, pag 2], who participated in the drafting of the rationale of the Civic Education Programs, citizenship is about "**living together in society** within a democratic framework of the Rule of Law and respect for rights in their fullest sense,? and with respect to its related values he mentions

"**justice and equity, and autonomy**, understood as the antithesis of authoritarianism or mere tutelage; tolerance and respect, and appreciation of diversity".

This author points out that citizenship may be visualized from the point of view of democracy, defining it as:

> A citizenship that fundamentally recognizes the equality of everybody?s rights and duties and a consequent mutual respect and respect for laws legitimately set forth. Individuals that practice citizenship are ideally conceived of as persons committed to democracy as the best form of government. This commitment implies considering citizenship as the best method for governing and settling differences and conflicts, through a legal and institutional framework that strengthens and allows society?s functioning as a whole. [5, pag 7].

Based on the concepts discussed related to the issue of citizenship, the following section presents a series of characteristics that Costa Rican citizens might possess, formulated by the authors.

Characterization of a citizen

Based on the above information, several characteristics are listed below which, according to the authors, good citizens must have; they are classified in four categories: living together, criticality, democracy, and social awareness.

Category	Indicator
Living together	1. Capacity for teamwork 2. Respectful and tolerant of different ways of thinking, beliefs, religions, gender diversity, and special needs, among others, of all other human beings. 3. Respectful of life in all its representations. 4. Capacity to resolve conflicts.
Criticality	5. Capacity to voice opinions on an issue or situation that affects society. 6. Capacity to look for viable solutions to a real problem. 7. Capacity to analyze data and results obtained with a real and objective perspective.
Democracy	8. Acknowledges and appreciates the democratic system as a need for living in society. 9. Knows and respects national legislation (laws, duties and rights). 10. Knows the mechanisms for defending their duties and rights.
Social awareness	11. Is aware of the reality of the country in the political, economic, social, and other spheres. 12. Participates in community and national development activities, concerning themselves about social problems and developing a sense of belonging. 13. Knows the country?s history, as well as issues related to their profession. 14. Shows solidarity with those who need it. 15. Is responsible when making decisions and acting.

Tabla 1.1: Characteristics of a good Costa Rican citizen (Note: this scheme is based on analysis of the Fundamental Education Law, No. 2160. (1957), Study programs in life-skills education. Third cycle of Basic General Education. Ministry of Public Education (2012). Study programs in Mathematics for Basic General Education and Diversified Cycle. San José, Costa Rica, and Montoya, L. (2010)[5]. Ética, Estética y Ciudadanía – Implementación y Abordaje de los Programas de Estudio de Educación Cívica, Artes Plásticas y Educación Musical. Fundamentación teórica [*Ethics, Aesthetics and Citizenship – Implementation and Approach of the Study Programs in Civic Education, Plastic Arts, and Music Education. Theoretical Rationale*].

Citizenship and mathematics

Costa Rica has recently experienced social changes in response to globalization, and the orientation of the educational system has therefore been revised and adjusted. The Ministry of Public Education (whose acronym in Spanish is MEP) has changed courses in some study programs, including Spanish for elementary education, Civic Education for secondary school, and Mathematics for elementary and secondary education.

The need to re-orient mathematics learning processes has been discussed on various occasions, working to provide a more human face of this discipline to students by including affective, ethical, attitudinal and sociocultural factors in its teaching (Castillo, 2010 [2]).

The new MEP Study Programs for Mathematics in elementary and secondary education were approved on May 21, 2012, and were intended to improve the basic formation of the Costa Rican population, through teaching mathematics in a way that would provide life skills (Ministerio de Educación Pública [MEP], 2012 [4]). In the specific case of mathematics and its role in society, the MEP [4, pag 18] states that the current study program:

> Is intended to establish a vocation of the mathematical competence especially associated with the construction of essential citizen capacities for the nation?s progress. It does not consist only of training minds to be able to carry out restricted activities such as mastering sophisticated demonstration techniques or building extremely abstract structures divorced from the environment, or for ethereal and private enjoyment of knowledge. Mathematics shall rather be used to improve citizens? understanding of and intervention in various physical, social, professional, scientific, and cultural spheres, providing them with capabilities that enable them to contribute to the nation?s progress with a spirit of responsibility and respect.

This text makes it evident that the MEP wants to change the way in which elementary and secondary education students are educated, so that an emphasis on cumbersome calculations and complicated procedures that never match reality will be replaced by one where mathematical abilities are used as tools to solve and analyze real day-to-day situations, thus contributing to the formation of behaviors or capacities associated with citizenship.

Another action leading to the formation of good citizens is the inclusion of themes which are relevant in many areas, not just mathematics, in the different courses.

For the MEP (2012, [4, pag 19]):

> The purpose of including these cross-cutting subjects is strongly furthered by the curricular approach that underpins the relationship of the Mathematics teaching-learning process with the social and cultural environments, which are naturally included in study plans.

The concept of citizenship in math education

Authors such as Vanegas [6] discuss citizenship development through math education. According to this author, this includes:

- Education in cross-cutting and trans-disciplinary values;
- Heightened political awareness and responsibility in constructing the identity and cohesion of curricular objectives;
- Constructing citizenship from a perspective of democratic communicative ethics;

- Constructing citizenship through a transformative and liberating dialogue;
- Leading towards democratic citizenship through solidary participatory math practices;
- Learning to form citizenship through math using exercises that seek a humanistic construction of math and its teaching.

Font, Giménez, Zorrilla and Larios [3, pag 61] indicate that, for instance, understanding citizenship as a competence in teacher formation, citizenship among mathematics teachers means "expressing a permanent conduct of respect for human beings' dignity, as an example of the development of a professional identity nourished by a set of values associated with ethical commitment in its practice"

Finally, the scheme prepared by the authors is presented on the concept of citizenship competence in math education.

In the most general of terms, citizenship in education will be understood as a general competence, which can be defined as follows: **The capacity of living harmoniously with other human beings, maintaining a critical respect for democracy, and being aware of the role of individuals in the development of a society with social and environmental awareness and defined values.**.

1.3 Methodological framework

This section summarizes the procedure used for collecting the data, as well as the instruments used and the analysis carried out.

Type of study and participants

This is a qualitative, specifically descriptive study, intended to describe the reasoning of trainee teachers when they carry out each one of the activities related to citizenship, and statistical analyses carried out in the Statistical Inference (MAB401) course, which was delivered during the fourth level of the Bachelor's and Licenciatura Degrees in Math Teaching at the Universidad Nacional de Costa Rica, in the first cycle of 2014. The field of work of these future teachers will be the Third Cycle of Basic General Education, and the Diversified Cycle (students between 12 and 17 years old).

The total enrollment in the course was 13 students, who are the participants in this research. The objective of the study was to determine the knowledge of these trainee teachers with respect to the characterization of a good citizen, to shed light on the concept of citizenship that can be taught through statistics and probabilities exercises, and how this concept can be developed in math teaching in secondary education.

Instruments and data collection

This cycle was carried out in three phases: verbal diagnosis, written diagnosis, and project preparation. The description of each is presented below.

1.3.1 Verbal diagnosis

A verbal diagnosis was made at the beginning of the course (February 2014) related to general elements on the role of mathematics in people?s formation, focusing principally on the importance of statistics and probability exercises in the formation of good citizens.

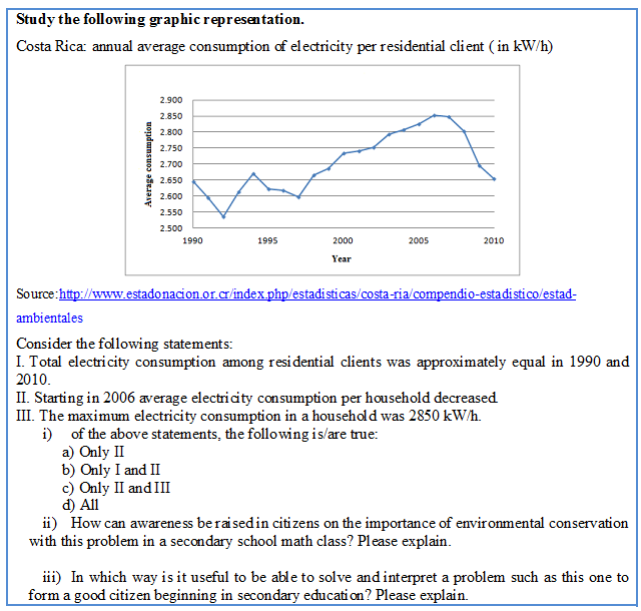
Specifically, trainee teachers were asked about their knowledge of the current MEP Mathematics Programs in secondary school and why mathematics is taught in secondary education. Questions were also asked about the meaning of being a good citizen, and how the areas of statistics and probability contribute to the formation of good citizens. Lastly, they were asked to think about whether it is possible to learn something about citizenship through mathematics, and how.

The method used in this activity was interrogation: the teacher in charge asked a question and provided an opportunity for students to give their opinion voluntarily; 18 questions were asked.

1.3.2 Written diagnosis

This activity was also carried out during the month of February 2014, and consisted in applying a questionnaire with six problems to be solved using statistical analysis. The objective of using this instrument was to obtain information about the initial perception of the group of trainee mathematics teachers about how statistics and probabilities problems may be used to form a good citizen beginning when students are in secondary education.

The problems presented included situations experienced in daily life by older persons, related to electricity consumption, diseases such as malaria and dengue, deaths in road accidents or caused by diseases, and vaccination. For every problem a content question was asked (related to statistics or probabilities), and then several questions were asked about solving these exercises using an educational approach. The following is an example of one of the activities presented.



Study the following graphic representation.

Costa Rica: annual average consumption of electricity per residential client (in kW/h)

Source: http://www.estadonacion.or.cr/index.php/estadisticas/costa-ria/compendio-estadistico/estad-ambientales

Consider the following statements:
I. Total electricity consumption among residential clients was approximately equal in 1990 and 2010.
II. Starting in 2006 average electricity consumption per household decreased.
III. The maximum electricity consumption in a household was 2850 kW/h.

i) of the above statements, the following is/are true:
a) Only II
b) Only I and II
c) Only II and III
d) All

ii) How can awareness be raised in citizens on the importance of environmental conservation with this problem in a secondary school math class? Please explain.

iii) In which way is it useful to be able to solve and interpret a problem such as this one to form a good citizen beginning in secondary education? Please explain.

Figura 1.1: Second problem situation. Written diagnosis, MAB401 course, I Cycle (2014).

1.3.3 Projects

The students, in groups of three or four members, carried out a project where they prepared a didactic unit about some content in the areas of probabilities or statistics included in the current MEP Mathematics Study Program, addressing the methodology of this program as well as the development of skills related to citizenship. This activity was implemented at the end of the MAB401 course, in June 2014.

Among other elements, each project was to include:

1. Activity presentation and development (problem situations) following the pedagogical mediation proposals in the MEP Mathematics Study Programs (lesson organization).

2. Answer to the question: How is citizenship developed in students through the proposed activities (problem situations)?

1.4 Data analysis

A detailed description was generated based on the data obtained in each of the activities carried out in the course, seeking to find similar ways of thinking among trainee teachers regarding the development of citizenship. Weaknesses in terms of abilities to use day-to-day activities for developing mathematics content related to citizen formation were also determined. The information from the verbal and written diagnoses was first presented, followed by the presentation of group work (Project). It is important to mention that when the opinion of trainee teachers is cited below, he or she is referred to as *student*, although the opinion reported is not of a single one of them, but rather the opinion most representative of the group as a whole.

1.4.1 Verbal diagnosis

When trainee teachers were asked about the importance of mathematics in society, general answers were elicited about applications in commerce and finances in daily life, which were usually correct. Students clearly pointed out the importance of mathematics as a reasoning structure, useful for solving problems beyond the mathematics content in itself. When asked about statistics and probabilities, answers were mostly aimed at applications that were familiar to them such as gambling, sports statistics, and car accidents, among others. For instance:

Example 1.1

Student[a]: *Perhaps for measuring the probability of succeeding in a project and, depending on which area it is developed in, what my probability of being successful is.*

[a]The answers of trainee teachers are cited as given by a "Student"; however, not all the quotations correspond to the same person.

With respect to the concept of a *good citizen*, trainee teachers indicated several of the duties associated with the practice of citizenship. For instance, abiding by the law, voting, or not littering. Slowly, the students began to express general citizen values: respect, persistence, responsibility and solidarity. According to trainee teachers, this might be conveyed in class, for instance, when attendance is called (responsibility), or when there were work groups (tolerance). It is worth noting that students indicated the need to create support groups for municipal governments to carry out common tasks regarding recycling and waste collection, among other activities.

When asked if mathematics education could contribute to forming good citizens, some respondents limited their contribution to financial education (investment, loans, etc.). Others were more critical, indicating that mathematics by itself does not necessarily contribute to form good citizens, because

mathematics may be learned without generating values, at least consciously. For instance:

Example 1.2

Student: *I think it can contribute, but to say that only teaching mathematics to a person makes him or her a better citizen is not correct. Yes, because it contributes to professional growth and in issues that require reasoning, it provides many benefits...but it cannot be said that a good citizen may be developed only through mathematics education.*

When asked specifically about statistics and probabilities in the formation of citizens, the power of these disciplines to convey information (in a descriptive form) to people was mentioned, as well as the possibility that these disciplines provide for making better decisions (optimization of resources, water management, waste management, contribution to social security, health and traffic).

Responses indicated that training in these disciplines is essential to be able to share relevant information about reality and to change it. For instance, to help people understand that water scarcity is not an eventual or intuitive matter, and that they should be educated to take care of water sources and existing water. They finally indicated the great responsibility of secondary educational institutions and therefore of their teachers.

Example 1.3

Student: *We are going to form persons, not only mathematically speaking, but we also have to instill values in those persons, not only go to the blackboard, apply a math exercise and that is it. No . . ., we also have to orient their formation responsibly.*

1.4.2 Written diagnosis

As indicated in the methodology, this diagnosis consisted of the implementation of six common activities. For the purposes of this investigation, one of them is analyzed here, in which it was possible to determine a stronger relationship with the concept of citizenship. Activity number 2 was selected, which was previously described in Figure (1.1).

With respect to the question associated with the statistical content itself, an important weakness was found in the interpretation of graphics, given that participants could not make the correct deduction.

When asked about the veracity of the statements, many of the trainee teachers indicated that all of them were correct, although it is evident that the first and third statements are not. In general, this indicates that the information was not properly analyzed, confirming that there are conceptual problems associated with the construction and interpretation of graphics, and measurements such as averages and maximum and minimum values.

The fundamental problem found was that given weaknesses in the mastery of contents associated with statistics, and the resulting problem of constructing erroneous concepts, it is difficult to evaluate the potential for providing competences related to citizenship, even though such potential may exist.

When asked *how may awareness be raised in citizens about the importance of environmental conservation with this problem in a high-school mathematics class,* students' statements were associated with the production of opinion-generating questions, as well as with trying to raise awareness about the resources that are consumed in the generation of electricity.

Example 1.4

Student:*The issue may be dealt with by taking a video or information with better examples of the impacts of excessive consumption of electricity on the environment, comparing statistical data, or otherwise by assigning students research on the subject, which will then be discussed in general terms, thus raising awareness..*

Although many students made observations about the importance of responsible use and actions to be taken, only a few of them clearly defined activities for their math class. For instance:

Example 1.5

Student: *By not using more than one electronic device at the same time, turning off lights during the day, using them only at night, when necessary.*

When the previous question is analyzed, it is important to note that the future mathematics teacher is making a judgment that is rather divorced from his or her profession and, although the statements he or she made may be useful, they show little capacity to associate their professional activity with the context. This type of contribution was prevalent in this group of trainee teachers.

In other cases, an important problem persisted at the mathematical and statistics level.

Example 1.6

Student: *With something as simple as the rule of three, it is possible to find out how much of the environment is going to be conserved or not in about 15 years.*

Finally, regarding usefulness of knowing how to solve and interpret a problem such as this to assist in forming good citizens starting in secondary education, many pointed out that mathematics education can raise awareness of electricity consumption, energy saving, and conservation of natural resources such as water.

Only one student indicated that this type of exercise may be used as a model to look for solutions to different problems, not only those related to excessive electricity consumption.

Example 1.7

Student: *If starting from secondary education individuals know how to solve and interpret problems such as this one, it would help them make comparisons between situations and the problems they generate,*

which will come to be useful in decision-making in their live; this helps form a citizen who is aware of current reality, and willing to participate in efforts to improve the country.

1.4.3 Projects

A total of four groups presented their didactic units, reporting on aspects related to the activities implemented in the unit and their relationship with citizenship competences. The work carried out by each of the groups is described below.

G1 Group

This group cited the MEP Study Program in terms of the relationship between the problems proposed in class and day-to-day life, specifically using the disciplines of statistics and probabilities.

> "The 21st Century requires persons who are able to understand, interpret and use information to understand reality, solve different problems and make intelligent decisions. The disciplines of Statistics and Probability are an everyday requirement to be able to understand what is happening in the world, and to act" (MEP, 2012 [4, pag 55]).

The students considered that it was important to form citizens with statistical abilities, and, in turn, that activities oriented to that end must be created.

Activity proposed by students of G1:

Group 1: "Assume that Peter and Charles buy a box that has 17 apples and each one of them pays half of the money to buy it, but when they are dividing the apples they notice that if each one of them takes 8 apples, there would be one extra, so they decide to flip a coin to avoid arguments about who will take the extra apple. What would be easier to obtain, heads or tails?"

The main reason for implementing this activity is taking into account that it is not possible to determine the best option, given that some groups would say that it is heads while others would say it is tails, depending on the tosses of each group.

G1 considers that this activity develops important values in the formation of citizens; specifically, they comment: *the problem proposed is related to conflict resolution, where those involved in the situation do not confront the problem in a violent manner, but instead look for an effective solution, leaving to randomness the decision of who will take the extra apple. Given the constant problems of violence, aggressions and similar problems experienced in society, an equally favorable situation for both parties would be a simple way for solving small debates without the need of having an opinion from a third person, or the possibility of worsening the situation.*

The activity carried out by this group may be classified in the category of **living together** described in Table 1.1 of the theoretical framework, as it promotes tolerance and conflict resolution in a peaceful manner. It should be emphasized that students propose a situation that is aimed at developing aspects that are cited in the MEP Study Program as gambling.

G2 Group

The general objective of its activity was to *instill environmental awareness in students by using graphics and tabulations for data analysis*.

This group proposes two activities under the general title **What do Costa Ricans do with wastes?** ,The first activity presented information about the *percentage of separation of wastes by provinces according to municipalities during 2012 and 2013* in a table, and then posed a series of questions related to data analysis and graphical representations that could be made. In the second activity, it was proposed that high-school students apply a questionnaire to all their classmates to find out whether recycling is carried out in their households, and the type of material recycled.

As they stated, with the activities they proposed G2 members are seeking to: *develop in the students one of the cross-cutting subjects proposed by the MEP - environmental culture for sustainable development.*

The work carried out by this group would fall under the category of **social awareness** in Table 1.1,since it deals with the subject of recycling and reducing the humans' global carbon footprint; through this activity they intend to make high-school students more aware of the reality in which they live, and therefore to behave in a more responsible way when making their decisions; in addition, subjects such as tabular summaries, graphic representations, and data analysis are addressed.

G3 Group

G3 members focused on the importance of statistics in day-to-day life. The objective of their activity was to *highlight the role of statistics as a tool for data analysis in different areas.*

The first part of the activity consisted of a short questionnaire that high school students had to apply to their relatives about the meaning of statistics and its use in their daily lives; the class would discuss the answers received when using the questionnaire. They also used statistics related to causes of fatal road accidents in order to develop the subject of graphic representation.

This same group proposed another activity related to the historical development of statistics in which student creativity was tested in the representation of historic fragments.

The members of this group considered that their activities complied with the guidelines set forth by Vanegas and Giménez (2011)[6] given that, among other things, they promote acknowledgement of mathematical procedures in real contexts, use historical elements, and foster cooperation.

The activities proposed by this group are classified under the category of **social awareness** in Table 1.1, as they lead to decision making in a responsible manner when they talk about the causes of traffic accidents; in addition, they consider history as a fundamental pillar to develop the topic of statistics, which is in line with the MEP Study Programs.

Grupo G4.

This group developed the statistical topics of quantitative variables and frequency distributions, and one of its objectives was to make conjectures about quantitative variables in daily life.

The activity proposed by this group was that high school students, in groups of five individuals, and with the help of some measuring instruments, should measure some part of the body of their peers,

creating a table and a graphic representation, and then share the results. This group considered that this activity contributed to developing *the right and willingness to participate in a community, through self-control, inclusive, peaceful and responsible actions, seeking to optimize people's well-being*.

After their participation in the proposed activities in their didactic unit, G4 members considered that these activities contributed to strengthen values by which students learn about group work, solidarity, respect for those around them, and therefore, citizenship; this would place these activities in the **social awareness** category, developing also topics such as representation of data in graphics and tables.

In general, students of the MAB401 course acknowledged the importance and need of developing abilities related to citizenship in high school classes; they considered that the cross-cutting topics guided the development of these abilities, which is in line with the guidelines of the MEP Study Programs.

The category which was mostly emphasized in the activities proposed by trainee mathematics teachers was **social awareness**, which is encouraging, since Costa Rican society is currently experiencing a social crisis in which such values have been fading away. Participants considered that it is important to instill values such as respect and tolerance, which should be part of the development of every citizen, and proposed that they should be promoted starting in secondary education in mathematics education, given that as teachers they should not only form students in theoretical concepts, but also provide them with tools that will allow them to evolve socially.

In brief, the four groups proposed problem activities that could be solved using statistics and probabilities knowledge that allowed high-school students to develop different abilities which are proposed in the MEP Mathematics Study Programs. The authors consider that this finding is positive as those in charge of implementing these activities are studying and do not have yet much experience in the creation of didactic units and contextualized problems. Furthermore, they focused their activities on developing citizenship beginning in Costa Rican middle school in aspects related to living together with other human beings and the environment.

It is considered that the activities created by trainee mathematics teachers - given that they do not yet have experience teaching in high-school classrooms, and that the methodology of the MEP Study Programs proposes a new form of teaching and learning math - did not elaborate on statistics and probability issues, but mainly focused on the preparation of tables and graphics, leaving aside the analysis of situations and decision making. In terms of citizenship development, some of their proposals were not as pertinent as needed, nor was it entirely clear how citizenship was actually promoted; they proposed situations where cooperative work is important, but left aside aspects related to social criticism and democracy.

The MAB401 course is taught at the fourth level (Bachelor's degree) of the Mathematics Teaching career, where the authors consider that trainee teachers should already have enough academic maturity to allow them to carry out better analysis, discussion and reasoning of the activities they proposed for teaching the topics of this course in secondary education, to elaborate and relate education in mathematics with the country's social, environmental, economic, and other contexts. They should not be satisfied by simply addressing a concept, but should also be able to relate several concepts with each other, with other courses, and with daily life.

It was expected that with the activities proposed, several probability and statistics concepts would be developed in more depth, as to allow students to analyze different situations, and not just represent data in a table or graphic. Regarding the concept of citizenship, it was expected that not only would the team work be reflected, but also that key subjects of Costa Rican society would have been addressed.

1.5 Conclusions

Among the main conclusions from the study carried out with trainee mathematics teachers in the MAB401 course, delivered during the fourth level of the Mathematics Teaching career offered by the Universidad Nacional de Costa Rica, the following stand out.

During the verbal diagnosis phase, when the trainee mathematics teachers were together in the classroom where the course was being delivered, the trainees had more intensive discussions of the aspects proposed in the different activities related to citizenship. This dynamic made it possible to have different opinions opposed or supported by others. In addition, the different points of view expressed were explained in the moment of their expression. This allowed trainee mathematics teachers to correctly express their ideas about what they thought a good citizen should be, as well as their ideas about the statistics and probability topics discussed..

Trainee teachers acknowledged the usefulness of mathematics education and, in particular, of statistics and probability, in the formation of good citizens. They indicated that mathematics teaching allows the development of reasoning skills and abilities, and the ability to analyze different situations in daily life. In particular, they focused on mentioning characteristics of a good citizen which were in line with the classification made by the authors in Table 1.1 of the theoretical framework. They also recognized the responsibility of mathematics education in secondary school to promote citizenship and, therefore, the commitment and duty that teachers have to raise social awareness among their students. .

During the written diagnosis, none of the trainee teachers was able to correctly interpret the graphic presented in the second activity 1.1about a basic statistical concept such as the average of a set of data. This could indicate that the basic topics of statistics and probabilities should be addressed in more detail in the statistics courses at the university. However, when the same activity was used to generate discussion of citizenship, the discussion, analysis and reasoning based on the information were more pertinent.

Regarding the projects carried out in groups, the proposed activities turned out to be interesting and acceptable, considering that it was one of the first experiences for future teachers in preparing didactic units based on the methodology established in the MEP Mathematics Study Programs, approved in 2012, in which they thought, perhaps for the first time, about how to promote citizenship through mathematics education. However, their capacities for reasoning and analysis on the value and potential of the activities mentioned are issues that remain to be discussed by the trainees and their teachers.

The creation of problem activities aimed at developing citizenship competences, as a first experience for future teachers, allowed discussion and a first approach to the true utility that mathematics educa-

tion should have in the construction of those competences.

In the projects implemented it was possible to see that most of the proposed activities for promoting citizenship among high school students were grouped in the category called **social awareness**, and some under the **living together** category, according to the scheme created by the authors in the theoretical framework. This may possibly be a result of an awareness of the need to solve some of the current needs of society, such as increased tolerance, solidarity with those who are in more need and vulnerable, respect for diversity in all its forms, love of the environment, contributing to the reduction of violence and bullying in primary and secondary school classrooms, and becoming more responsible in various actions and decision making; however, other categories, such as **criticality** and **democracy** were not taken into consideration.

Finally, it is important to mention that future teachers have important conceptual weaknesses about probability, statistics and mathematics which hinder their ability to create more sophisticated activities for teaching the topics and promoting citizenship among high school students. This is an alert for the university, about the need to analyze how future high school teachers are being trained; it is desirable that these fourth-level students in the Mathematics Teaching career have a greater capacity for analysis that would allow them to have a clear idea about the objective of mathematics education in secondary school, and have the necessary tools for creating didactic situations aimed at developing the necessary competences among their high school students.

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